Quantum walks as a quantum simulators

Iván Márquez Martín
23/10/2019

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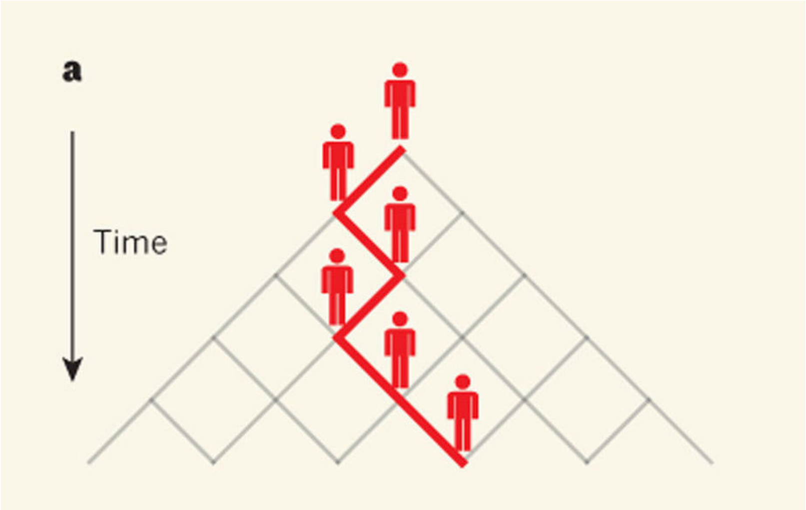
I. Introduction to QWs
II. From QWs to Dirac equation
III. QWs in hexagonal and triangular lattices.

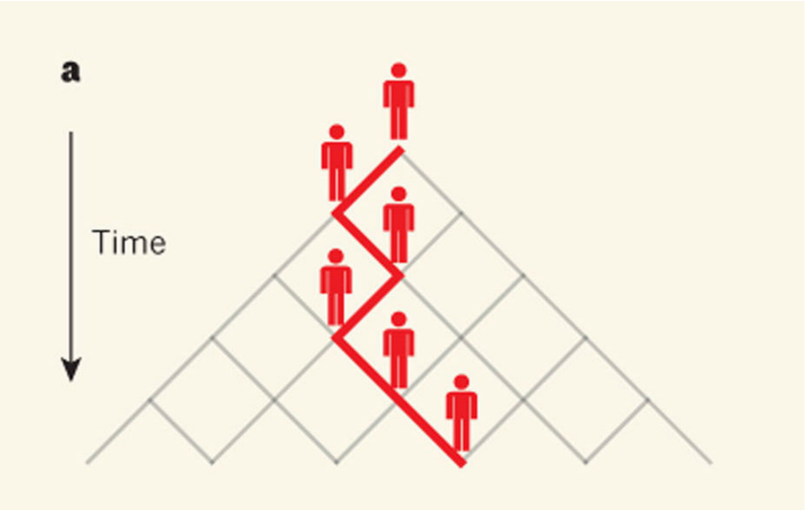
I. Introduction to QW

In the <u>CRW</u>



Same prob. to move left or right

I. Introduction to QW



Hilbert space $\mathcal{H} = \mathcal{H}_p \otimes \mathcal{H}_c$

$\Psi \in \mathcal{H}$

- $\mathcal{H}_p$ position sites $|i\rangle$ canonical basis $i \in \mathbb{Z}$
- $\mathcal{H}_c$ coin state $|c\rangle$ canonical basis of $\mathcal{H}_c$ $c \in \uparrow, \downarrow$

I. Introduction to QW

Unitary operator $U = S(\mathbb{I} \otimes C)$

C: coin operator

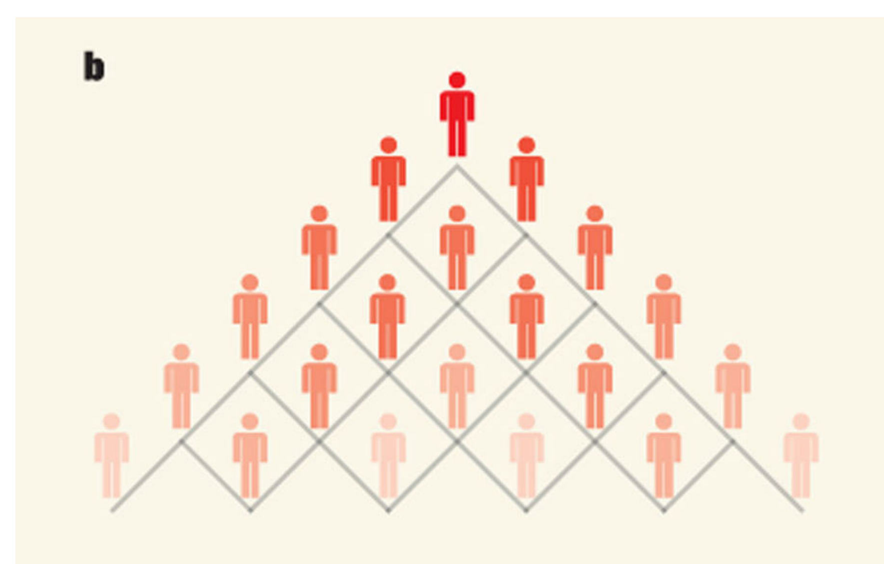
$$C = \begin{pmatrix} \cos\theta & \mathrm{i}\sin\theta \\ \mathrm{i}\sin\theta & \cos\theta \end{pmatrix} \qquad \begin{aligned} C &: |\uparrow\rangle \to \cos\theta\,|\uparrow\rangle + \mathrm{i}\sin\theta\,|\downarrow\rangle \\ C &: |\downarrow\rangle \to \mathrm{i}\sin\theta\,|\uparrow\rangle + \cos\theta\,|\downarrow\rangle \end{aligned}$$

Superposition

S: shift operator

$$S = \sum_{i\in\mathbb{Z}} |i+1\rangle\langle i| \otimes |\uparrow\rangle\langle\uparrow| + |i-1\rangle\langle i| \otimes |\downarrow\rangle\langle\downarrow|$$

Time



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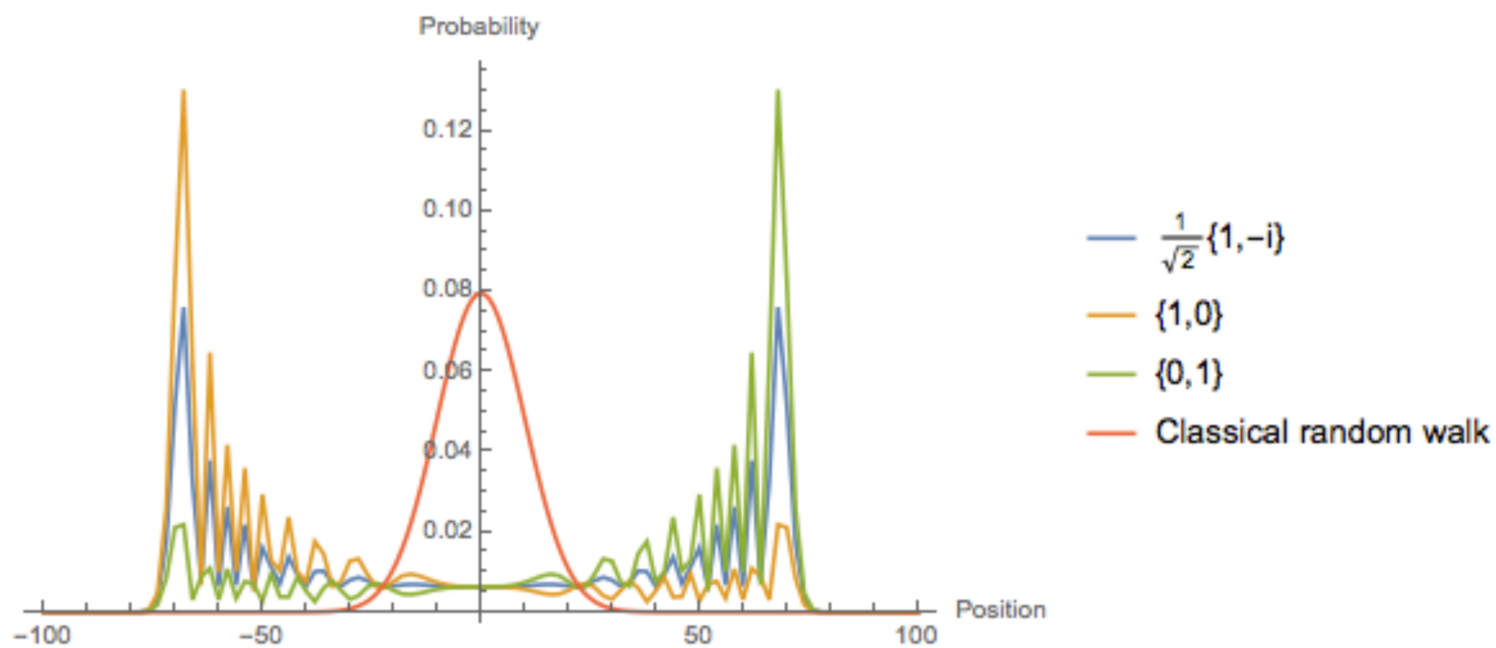
I. Introduction to QW

Probability density $P(i;t) = |\Psi_{t,i}|^2$



Spreading faster than CRW

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I. Introduction to QW

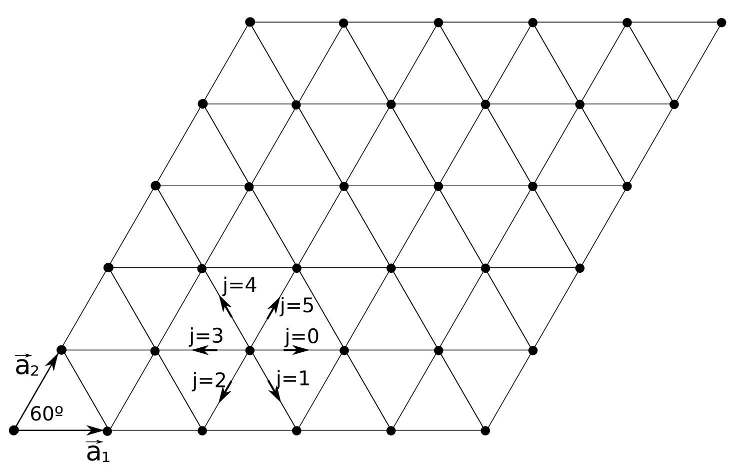
Quantum algorithm

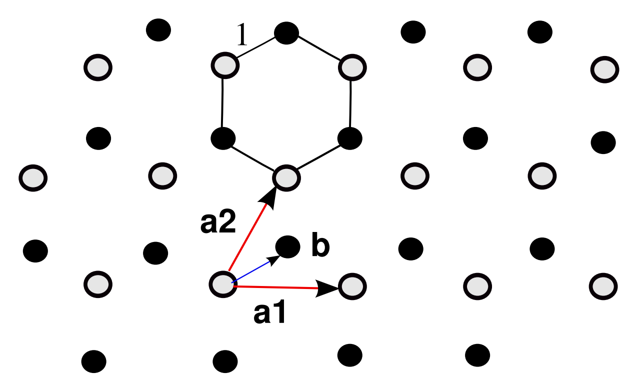
e.g. Searching in graphs

first search algorithm in a hypercube $\mathcal{O}(\sqrt{N})$

Quantum random-walk search algorithm, Phys. Rev. A 67, 052307 (2003)

In 2D time complexity $\mathcal{O}(\sqrt{N \log N})$





G. Abal, R. Donangelo, F. L. Marquezino, et al. “Spatial search on a honeycomb network”.

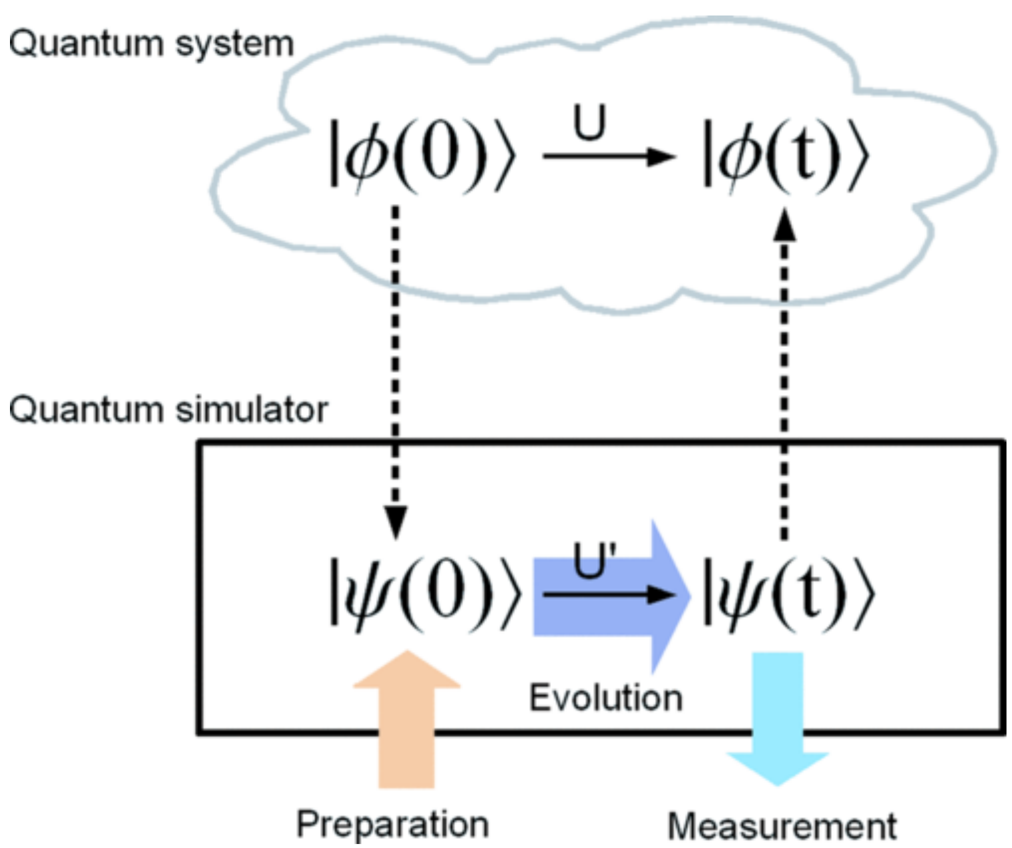
In: Mathematical Structures in Computer Science 20.6 (2010), pp. 999–1009

G. Abal, R. Donangelo, M. Forets, et al. “Spatial quantum search in a triangular network”.

In: Mathematical Structures in Computer Science 22.3 (2012), pp. 521–531. issn: 09601295.

I. Introduction to QW

Quantum simulation



I. M. Georgescu, S. Ashhab, and Franco Nori
Rev. Mod. Phys. **86**, 153

TABLE II. Potential applications of quantum simulators and the physical systems in which they could be implemented, along with relevant references. We note that this is not an exhaustive list.

Application		Proposed implementation
Condensed-matter physics:		
	Hubbard models	Atoms (Jaksch *et al.*, 1998; Greiner *et al.*, 2002)*
		Ions (Deng, Porras, and Cirac, 2008)
		Polar molecules (Ortner *et al.*, 2009)
		Quantum dots (Byrnes *et al.*, 2008)
		Cavities (Greentree *et al.*, 2006; Hartmann, Brandao, and Plenio, 2006; Angelakis, Santos, and Bose, 2007)
	Spin models	Atoms (Jané *et al.*, 2003; Garcia-Ripoll, Martin-Delgado, and Cirac, 2004, Simon *et al.*, 2011; Struck *et al.*, 2011)*
		Ions (Jané *et al.*, 2003; Porras and Cirac, 2004b; Deng, Porras, and Cirac, 2005; Bermudez, Porras, and Martin-Delgado, 2009; Edwards *et al.*, 2010; Lanyon *et al.*, 2011*; Kim *et al.*, 2011; Britton *et al.*, 2012)*
		Cavities (Cho, Angelakis, and Bose, 2008a; Chen *et al.*, 2010)
		Nuclear spins on diamond surface (Cai *et al.*, 2013)
		Superconducting circuits (Tsokomos, Ashhab, and Nori, 2010)
		Electrons on helium (Mostame and Schützhold, 2008)
	Quantum phase transitions	Atoms (Greiner *et al.*, 2002)*
		Polar molecules (Capogrosso-Sansone *et al.*, 2010; Pollet *et al.*, 2010)
		Ions (Retzker *et al.*, 2008; Friedenauer *et al.*, 2008)*; Ivanov *et al.*, 2009; Giorgi, Paganelli, and Galve, 2010
		NMR (Peng, Du, and Suter, 2005; Roumpos, Master, and Yamamoto, 2007; Zhang *et al.*, 2008)
		Superconducting circuits (van Oudenaarden and Mooij, 1996)*
	Spin glasses	DQS (Lidar and Biham, 1997)
		Superconducting circuits (Tsomokos, Ashhab, and Nori, 2008)
	Disordered systems	Atoms (Schulte *et al.*, 2005; Fallani *et al.*, 2007*; Billy *et al.*, 2008; Roati *et al.*, 2008)*
		Ions (Bermudez, Martin-Delgado, and Porras, 2010)
		Superconducting circuits (Garcia-Ripoll, Solano, and Martin-Delgado, 2008)
		NMR (Álvarez and Suter, 2010; Banerjee *et al.*, 2013)*
	Frustrated systems	Ions (Porras and Cirac, 2006b; Kim *et al.*, 2010)*
		Photons (Ma *et al.*, 2011)*
	High-T_c superconductivity	DQS (Yamaguchi and Yamamoto, 2002)
		Quantum dots (Manousakis, 2002)
	BCS pairing	NMR (Yang *et al.*, 2006)*
	BCS-BEC crossover	Atoms (Regal, Greiner, and Jin, 2004; Zwierlein *et al.*, 2005)*
	Metamaterials	Superconducting circuits (Rakhmanov *et al.*, 2008)
	Time-symmetry breaking	Superconducting circuits (Koch *et al.*, 2010)
	Topological order	Atoms (Aguado *et al.*, 2008)
		Polar molecules (Micheli, Brennen, and Zoller, 2006)
		Linear optics (Lu *et al.*, 2009)*
		Superconducting circuits (You *et al.*, 2010)

*Experimental realizations.

TABLE III. Continuation of Table II, but focused on applications other than condensed-matter physics. As in Table II, this is not an exhaustive list.

Application		Proposed implementation
High-energy physics:		
	Lattice gauge theories	DQS (Byrnes and Yamamoto, 2006)
		Atoms (Büchler *et al.*, 2005)
	Dirac particles	Ions (Lamata *et al.*, 2007; Casanova *et al.*, 2010, 2011; Gerritsma *et al.*, 2010*; Rusin and Zawadzki, 2010)
		Atoms (Goldman *et al.*, 2009; Hou, Yang, and Liu, 2009; Cirac, Maraner, and Pachos, 2010)
	Nucleons	Photons (Semiao and Paternostro, 2012)
Cosmology:		
	Unruh effect	Ions (Alsing, Dowling, and Milburn, 2005)
	Hawking radiation	Atoms (Giovanazzi, 2005)
		Ions (Horstmann *et al.*, 2010)
		Superconducting circuits (Nation *et al.*, 2009)
	Universe expansion	BEC (Fischer and Schützhold, 2004)
		Ions (Schützhold and Mostame, 2005; Menicucci, Olson, and Milburn, 2010)
Atomic physics:		
	Cavity QED	Superconducting circuits (You and Nori, 2003; Wallraff *et al.*, 2004)*
	Cooling	Superconducting circuits (Grajcar *et al.*, 2008)*; You and Nori, 2011)
Open systems:		
		NMR (Tseng *et al.*, 2000)*
		Ions (Piilo and Maniscalco, 2006; Barreiro *et al.*, 2011)*
		Superconducting circuits (Li *et al.*, 2013)*
Chemistry:		
	Thermal rate calculations	DQS (Lidar and Wang, 1999)
	Molecular energies	DQS (Aspuru-Guzik *et al.*, 2005)
		Linear optics (Lanyon *et al.*, 2010)*
		NMR (Du *et al.*, 2010)*
	Chemical reactions	DQS (Kassal *et al.*, 2008)
		Quantum dots (Smirnov *et al.*, 2007)
Quantum chaos:		
		NMR (Weinstein *et al.*, 2002)*
		Linear optics (Howell and Yeaze, 1999)
Interferometry:		
		Ions (Leibfried *et al.*, 2002*; Hu, Feng, and Lee, 2012; Lau and James, 2012)
		Photons (Aaronson and Arkhipov, 2011; Broome *et al.*, 2013*; Crespi *et al.*, 2013; Spring *et al.*, 2013; Tillmann *et al.*, 2013)*
		Superconducting circuits (Zhou, Dong *et al.*, 2008; Liao *et al.*, 2010)
Other applications:		
	Schrödinger equation	DQS (Boghosian and Taylor, 1998a)
	Quantum thermodynamics	Superconducting circuits (Quan *et al.*, 2006, 2007)

*Experimental realizations.

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II. From QWs to Dirac equation

III. QWs in hexagonal and triangular lattices.

II. From QWs to Dirac equation

Unitary operator $U = S(\mathbb{I} \otimes C)$

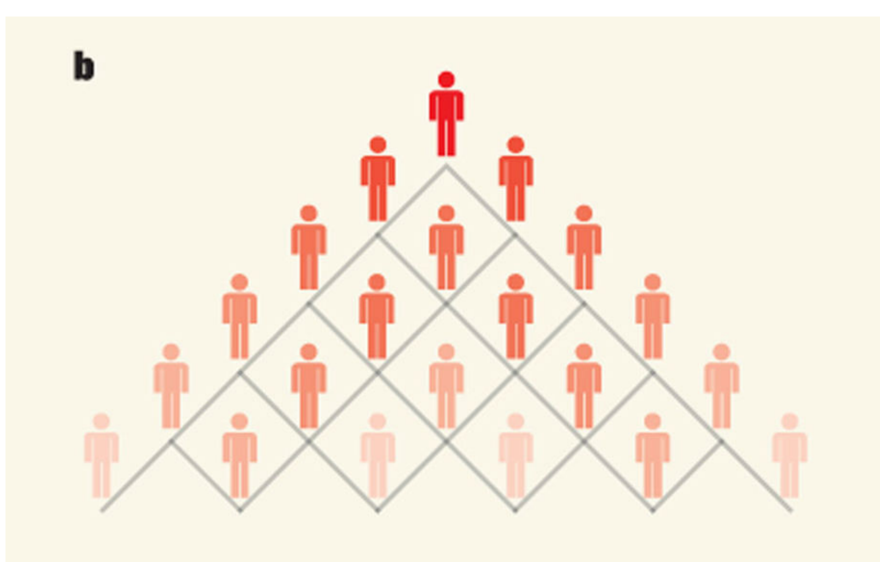
C: coin operator
$$C = \begin{pmatrix} \cos\theta & \mathrm{i}\sin\theta \\ \mathrm{i}\sin\theta & \cos\theta \end{pmatrix}$$

S: shift operator
$$S = \sum_{i \in \mathbb{Z}} |i+\epsilon\rangle\langle i| \otimes |\uparrow\rangle\langle\uparrow| + |i-\epsilon\rangle\langle i| \otimes |\downarrow\rangle\langle\downarrow|$$

The finite difference eqs:
$$\Psi^{\uparrow}_{t+1,i} = \cos\theta\Psi^{\uparrow}_{t,i+\epsilon} + \mathrm{i}\sin\theta\Psi^{\downarrow}_{t,i+\epsilon}$$
$$\Psi^{\downarrow}_{t+1,i} = \mathrm{i}\sin\theta\Psi^{\uparrow}_{t,i-\epsilon} + \cos\theta\Psi^{\downarrow}_{t,i-\epsilon}$$

$$\theta = \epsilon m$$

Time ϵ



Continuous limit
$$\epsilon \longrightarrow 0$$

$$\partial_t\Psi^{\uparrow} = -\partial_i\Psi^{\uparrow} - \mathrm{i}m\Psi^{\downarrow}$$
$$\partial_t\Psi^{\downarrow} = \partial_i\Psi^{\downarrow} - \mathrm{i}m\Psi^{\uparrow}$$

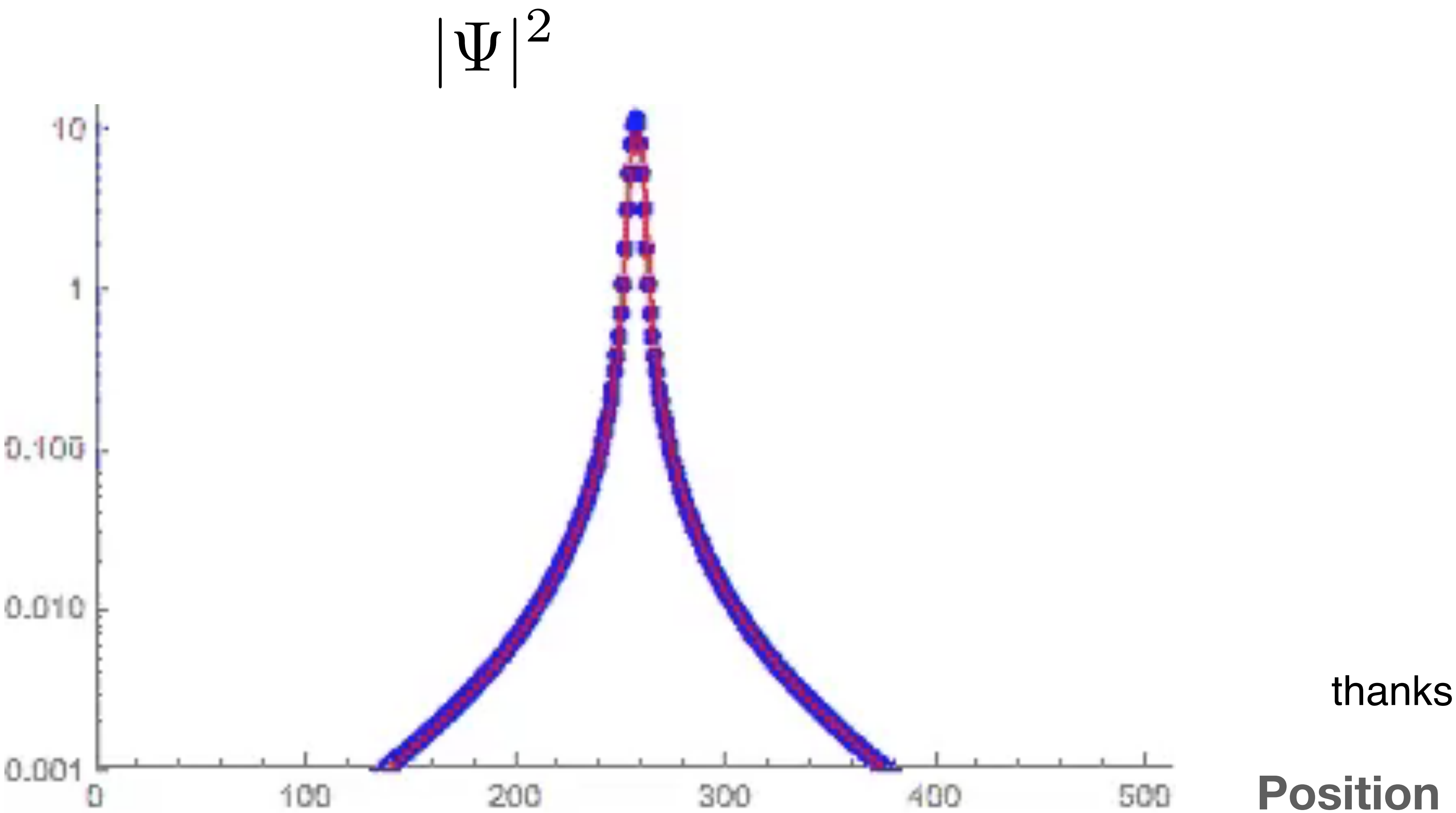
II. From QWs to Dirac equation

- Relativistic wave equation which describes all spin 1/2 particles
- First theory that combines fully SR and QM

$\theta = \epsilon \; m = 0.24$
$m = 20$

$$i\partial_t \Psi = i(\sigma_z \partial_x - i\sigma_x m)\Psi$$

QW



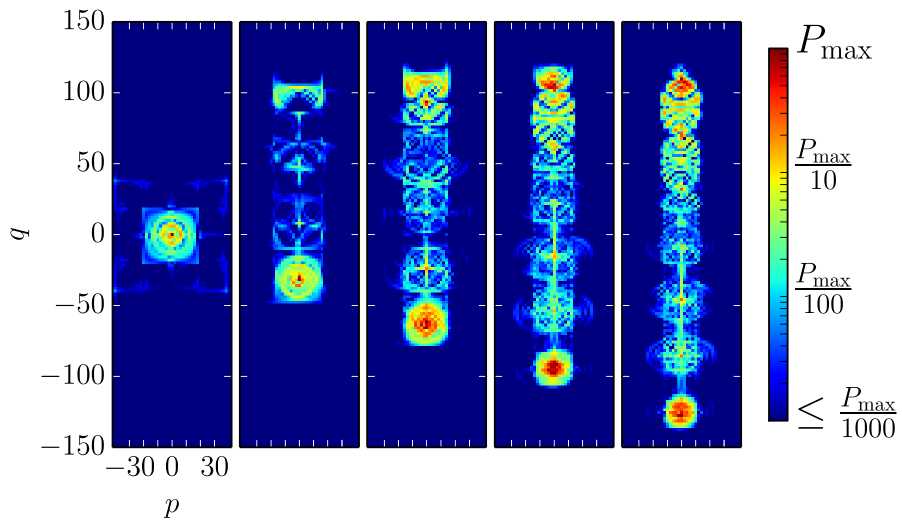
thanks to Giuseppe Di Molfetta

II. From QWs to Dirac equation

QWs can simulate processes that are very complicated to observe experimentally

Gauge fields

G. Di Molfetta and F. Debbasch. "Discrete-time quantum walks: Continuous limit symmetries". In: *Journal of Mathematical Physics* 53.12 (2012)



Pablo Arnault and Fabrice Debbasch. "Quantum walks and discrete gauge theories". In: *Physical Review A - Atomic, Molecular, and Optical Physics* 93.5 (2016), pp. 1–6.

$E \times B$ drift

- Pablo Arnault, Giuseppe Di Molfetta, Marc Brachet, et al. "Quantum walks and non-Abelian discrete gauge theory". In: *Physical Review A* 94.1 (2016)

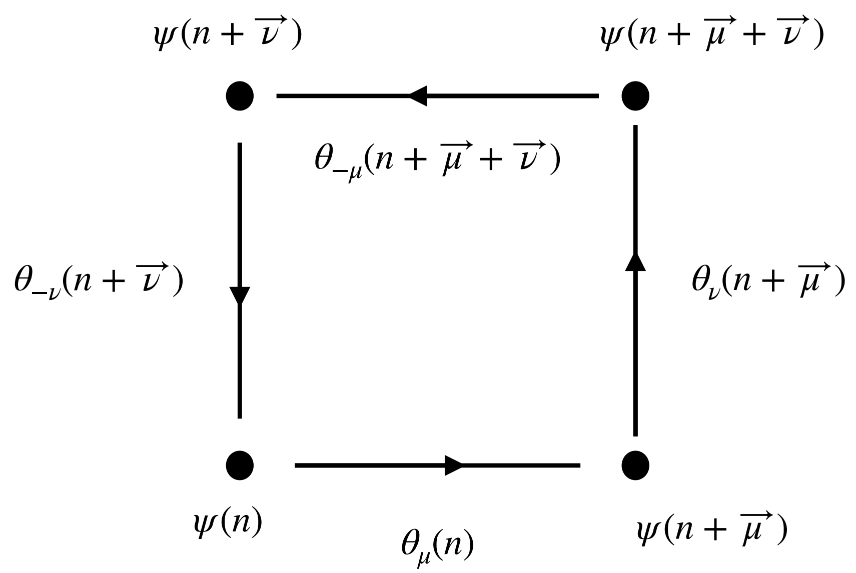
II. From QWs to Dirac equation

Gauge invariance in DTQW

Pablo Arnault and Fabrice Debbasch. "Quantum walks and discrete gauge theories". In: *Physical Review A - Atomic, Molecular, and Optical Physics* 93.5 (2016), pp. 1–6.

Electromagnetic lattice gauge invariance in two-dimensional discrete-time quantum walks
Iván Márquez-Martín, Pablo Arnault, Giuseppe Di Molfetta, and Armando Pérez. Phys. Rev. A 98, 032333 (2018)

Close analogies to LGT



C. Cedzich, T. Geib, A. H. Werner, et al. "Quantum walks in external gauge fields". In: *Journal of Mathematical Physics* 60.1 (2019)

Unified framework to understand U(1) gauge invariance, in DTQW with coin spaces of arbitrary dimensions

II. From QWs to Dirac equation

QWs can simulate processes that are very complicated to observe experimentally

Zitterbewegung → rapid oscillation of Dirac particles around their trajectories

Quantum walks and quantum simulations with Bloch-oscillating spinor atoms D. Witthaut Phys. Rev. A **82**, 033602
(and more)

Quantum simulation of the Dirac equation. R. Gerritsma, G. Kirchmair, F. Zähringer, E. Solano, R. Blatt and C.F. Roos, Nature 463, 68 (2010)

Dirac equation in curved space-time

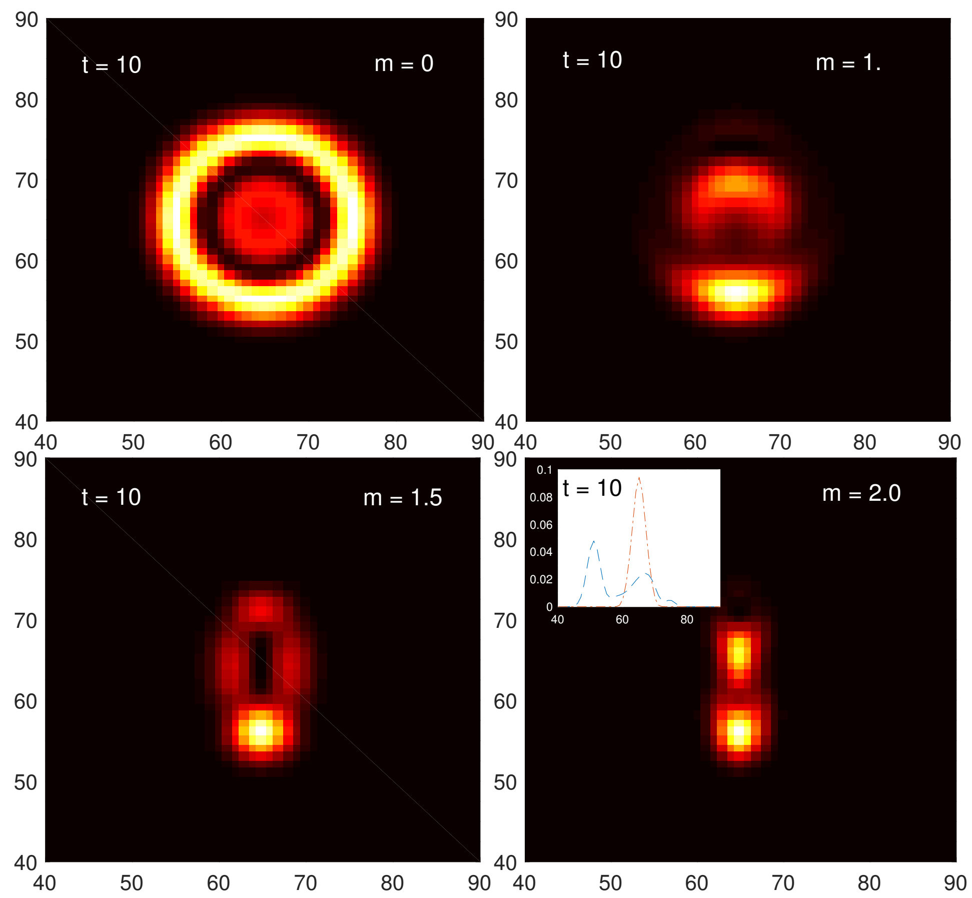
Giuseppe Di Molfetta, Marc Brachet, and Fabrice Debbasch. Quantum walks in artificial electric and gravitational fields. *Physica A: Statistical Mechanics and its Applications*, 397:157–168, 2014.

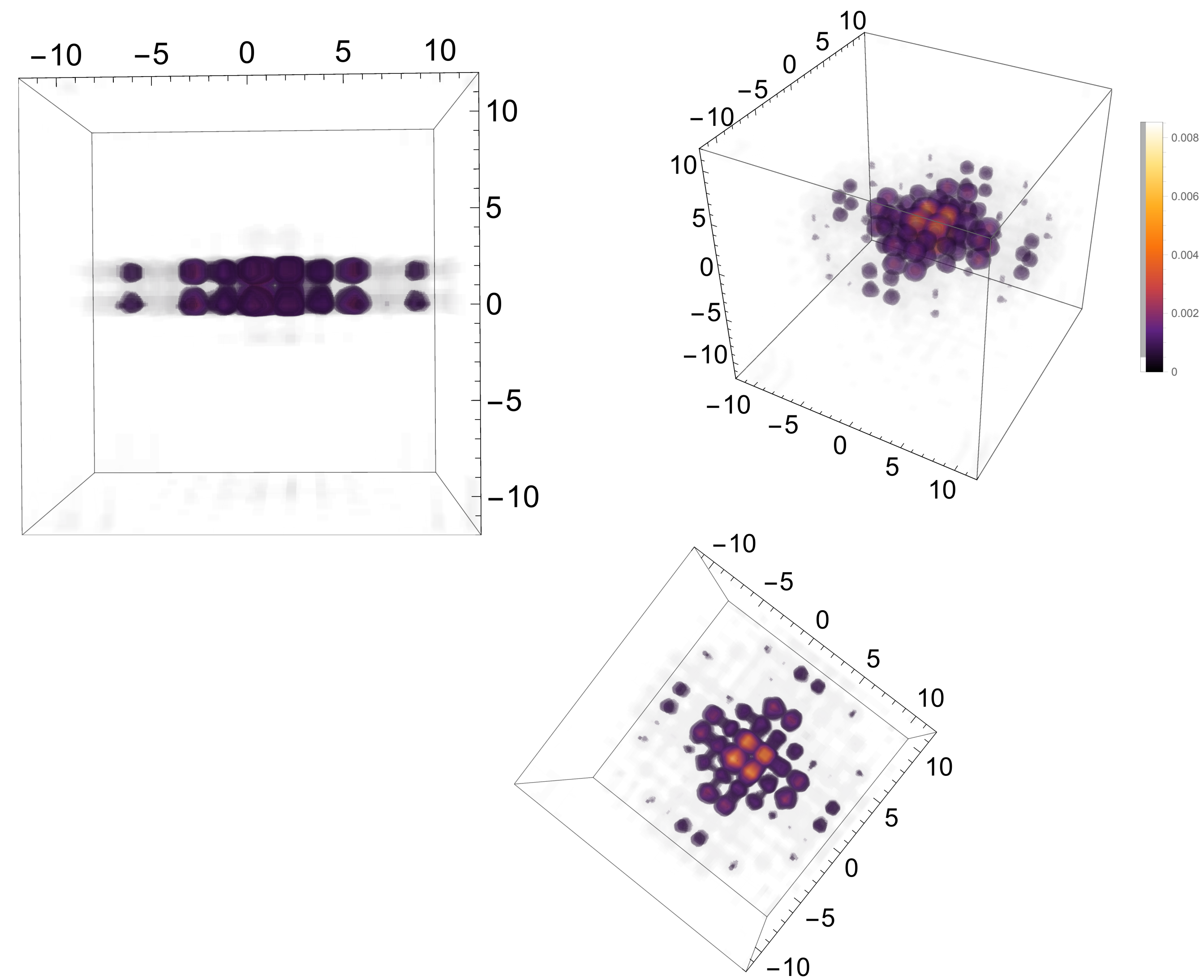
P. Arrighi and F. Facchini. Quantum walking in curved space-time: (3+1) dimensions, and beyond. *Quantum Information and Computation*, 17(9-10):0810–0824, 2017

Pablo Arnault and Fabrice Debbasch. “Quantum walks and gravitational waves”. In: *Annals of Physics* 383 (2017),

II. From QWs to Dirac equation

Brane theories → new extra physical dimensions take into account. a few TeV to 10^16 TeV





I.Márquez-Martín, G. Di Molfetta and A. Pérez, Phys. Rev. A 95 042112 (2017)

II. From QWs to Dirac equation

QWs allow us to study in a deeper way physical models, i.e. relativistic particles

from discrete to continuous

This approach could potentially be used to understand wider theories such as Quantum field theory, Quantum Gravity…

A quantum cellular automaton for one-dimensional QED.
P.Arrighi, C.Bény, T.Farrely arXiv:1903.07007

What happens if we change the topology of the lattice?

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I. Introduction to QWs
II. From QWs to Dirac equation
III. QWs in hexagonal and triangular lattices.

III. Hexagonal and triangular QWs

Topological phases

Takuya Kitagawa, Mark S. Rudner, Erez Berg, et al. “Exploring topological phases with quantum walks”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 82.3 (2010)

Search algorithms

G. Abal, R. Donangelo, F. L. Marquezino, et al. “Spatial search on a honeycomb network”.

In: Mathematical Structures in Computer Science 20.6 (2010), pp. 999–1009

G. Abal, R. Donangelo, M. Forets, et al. “Spatial quantum search in a triangular network”.

In: Mathematical Structures in Computer Science 22.3 (2012), pp. 521–531. issn: 09601295.

Localization processes →

P. W. Anderson. “Absence of Diffusion in Certain Random Lattices”.Phys. Rev. 109 (1956)

Changyuan Lyu, Luyan Yu, and Shengjun Wu. “Localization in quantum walks on a honeycomb network”. In: *Physical Review A - Atomic, Molecular, and Optical Physics* 92.5 (2015)

III. Hexagonal and triangular QWs

Graphene

Dirac-like electronic structure and dynamic

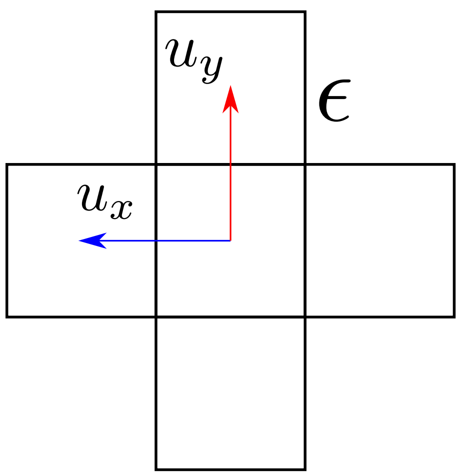
Possible physical implementation of QWs

Ioannis G. Karafyllidis. “Quantum walks on graphene nanoribbons using quantum gates as coins”. In: *Journal of Computational Science* 11 (2015)

Hamza Bougroura, Habib Aissaoui, Nicholas Chancellor, et al. “Quantum- walk transport properties on graphene structures”. In: *Physical Review A* 94.6 (2016)

III. Hexagonal and triangular grid QWs

2D QW in the grid



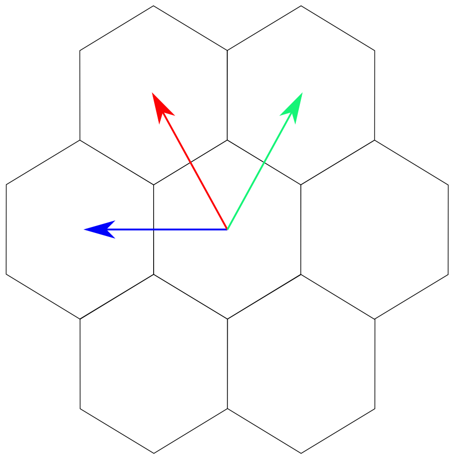
Continuous limit

$$\epsilon \to 0$$

Dirac Hamiltonian 2D

$$H_D = p_x\sigma_x + p_y\sigma_y + m\sigma_z$$

2D QW in a hexagonal lattice



$$u_i = \cos\left(i\frac{2\pi}{3}\right)u_x + \sin\left(i\frac{2\pi}{3}\right)u_y$$

$$i = 0, 1, 2$$

Continuous limit

$$\epsilon \to 0$$

Equivalent Dirac Hamiltonian 2D

$$H'_D = \sum_i \pi_i \tau_i + m\sigma_z$$

$$H'_D = H_D$$

Momentum along *i* direction

$$\pi_i = u_i^\nu p_\nu$$

New unitary matrices

$$\nu = 1, 2$$

III. Hexagonal and triangular QWs

In order to find the τ_i matrices

i) $\sum_i \pi_i \tau_i = \sigma_x p_x + \sigma_y p_y$

ii) eigenvalues $\{1, -1\}$

$$\tau_i = U_i^\dagger \sigma_z U_i$$

→

Unique solution, up to a sign

$$\tau_0 = \frac{2}{3}\sigma_x + \xi\sigma_z$$

$$\tau_1 = -\frac{1}{3}\sigma_x + \frac{\sqrt{3}}{3}\sigma_y + \xi\sigma_z$$

$$\tau_2 = -\frac{1}{3}\sigma_x - \frac{\sqrt{3}}{3}\sigma_y + \xi\sigma_z$$

$$\xi = \pm\frac{\sqrt{5}}{3}$$

III. Hexagonal and triangular QWs

Let's define a QW over this hexagonal grid such a in the continuous limit recovers H_D

Evolution of a state $$|\psi(t+\epsilon)\rangle = e^{-i\epsilon H_D} |\psi(t)\rangle$$

$$H_D = \sum_i \pi_i \tau_i + m\sigma_z$$

Using Lie-Trotter product formula

$$e^{-\mathrm{i}\epsilon\left(\sum_i \pi_i\tau_i + m\sigma_z\right)} \approx \prod_{i=0}^{2} e^{-\mathrm{i}\epsilon m\sigma_z} e^{-\mathrm{i}\epsilon\pi_i\tau_i}$$

removing second order epsilon terms

III. Hexagonal and triangular QWs

Making use of the condition that the eigenvalues are $\{1, -1\}$

$$e^{-\mathrm{i}\epsilon\tau_i\pi_i} = e^{-\mathrm{i}\epsilon U_i^\dagger \sigma_z U_i \pi_i} = U_i^\dagger e^{-\mathrm{i}\epsilon\sigma_z\pi_i} U_i = U_i^\dagger T_{i,\epsilon} U_i$$

where the translation operator along u_i

III. Hexagonal and triangular QWs

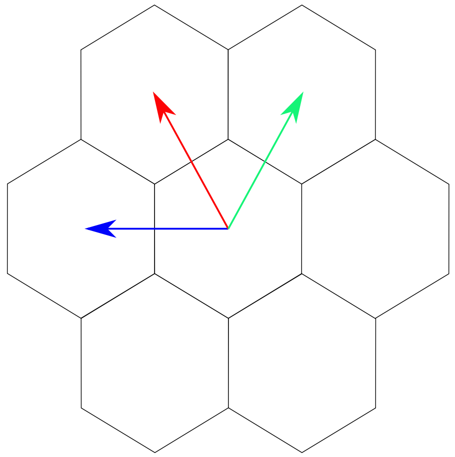
Finally our QW over the hexagonal lattice is given by:

$$|\widetilde{\psi}(t+\varepsilon)\rangle = (WT_{2,\varepsilon}WT_{1,\varepsilon}WT_{0,\varepsilon})\,|\widetilde{\psi}(t)\rangle$$

$$W = U_0 S U_0^\dagger M \qquad |\widetilde{\psi}(t)\rangle \equiv U_0\,|\psi(t)\rangle$$

$$S = e^{\mathrm{i}\frac{\pi}{3}}\mathcal{R}_{\sigma_z}(\frac{2\pi}{3})$$

By construction, Dirac eq is recover when $\epsilon \to 0$

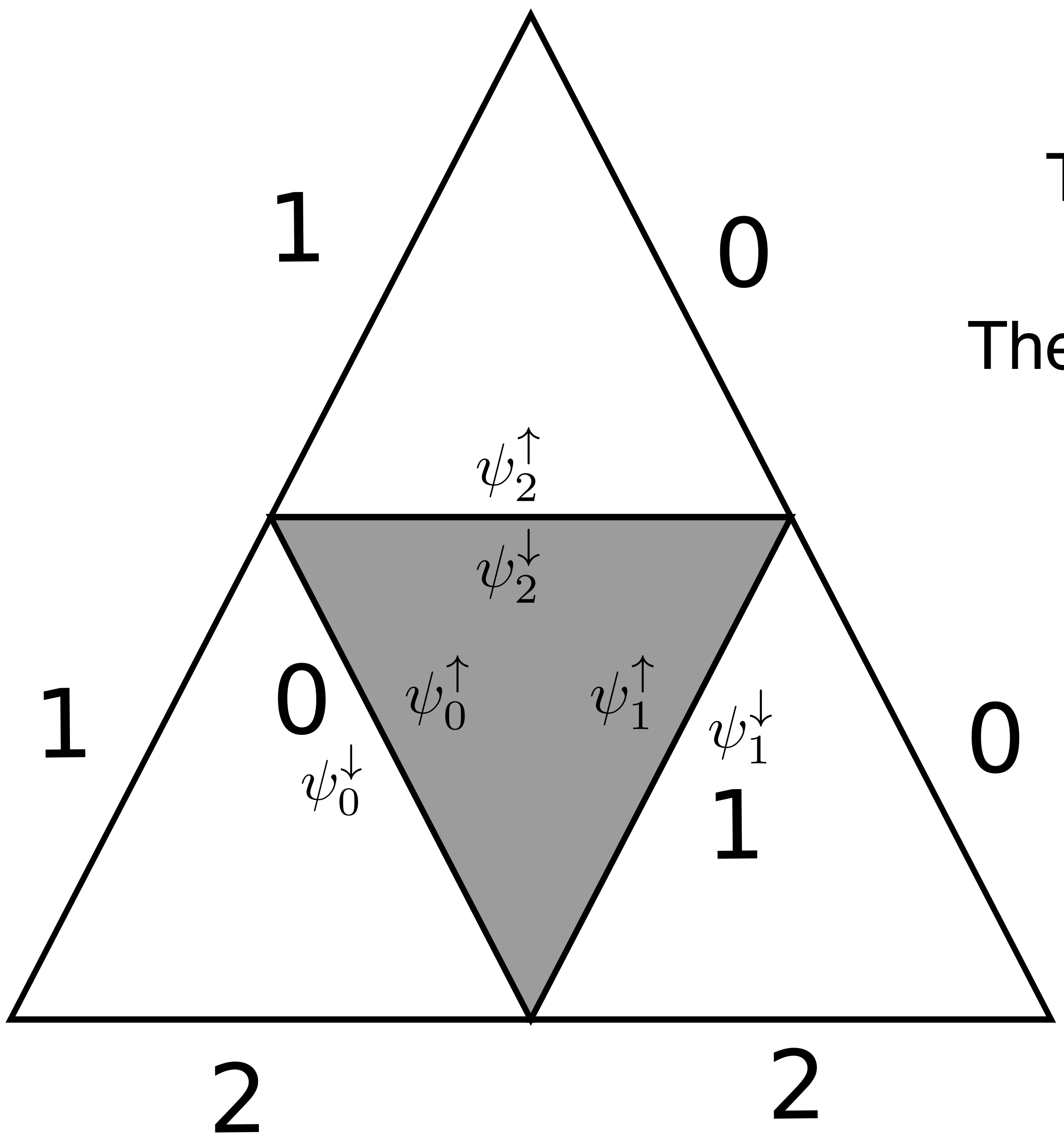


Having understood the honeycomb QW, it will be easier to tackle the triangular case

P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. The Dirac equation as a quantum walk over the honeycomb and triangular lattices. Phys. Rev. A 97, 062111

III. Hexagonal and triangular QWs

Triangular QW

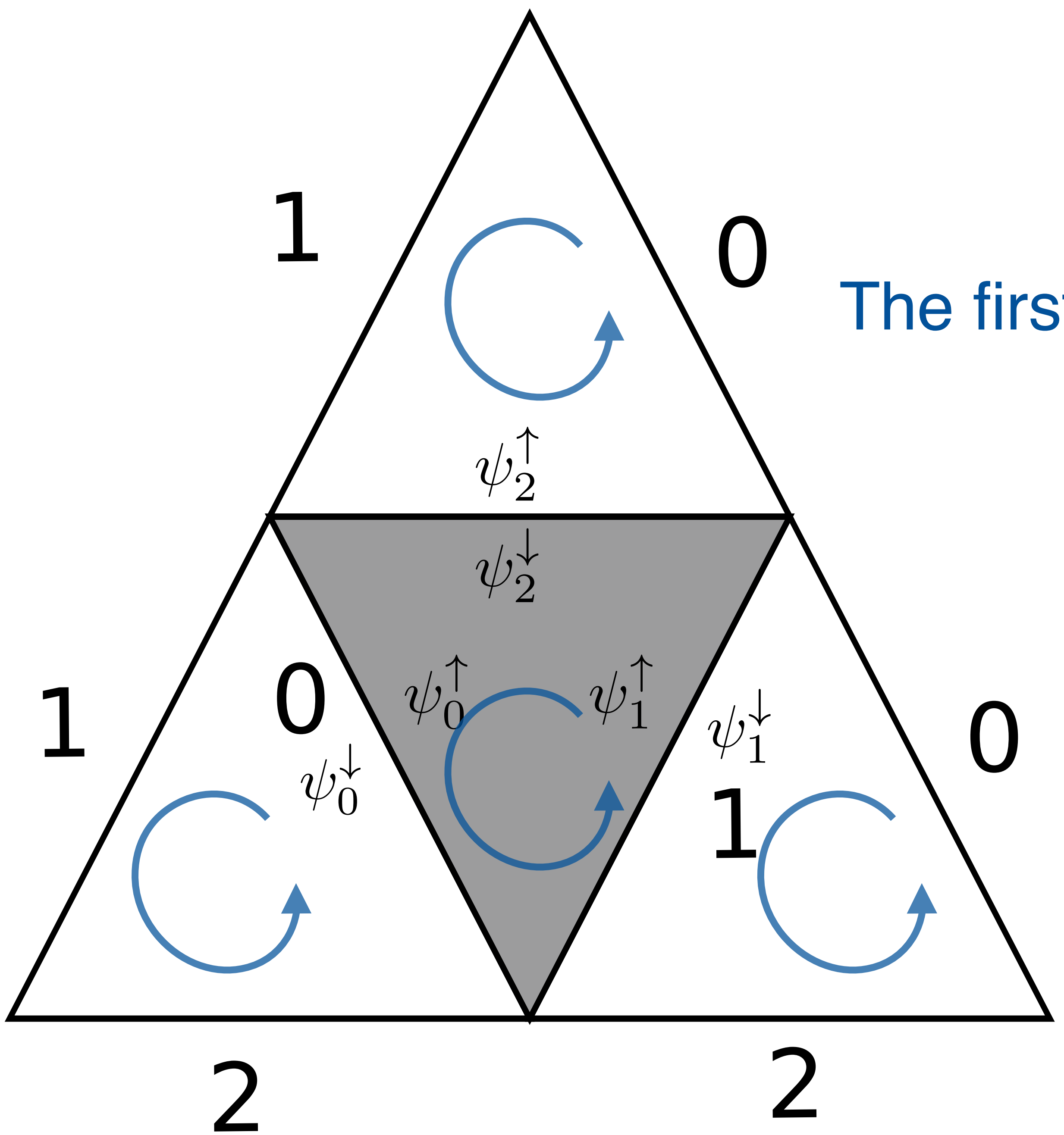


The walker lies on the edges

The dynamics: composition of two operators

III. Hexagonal and triangular QWs

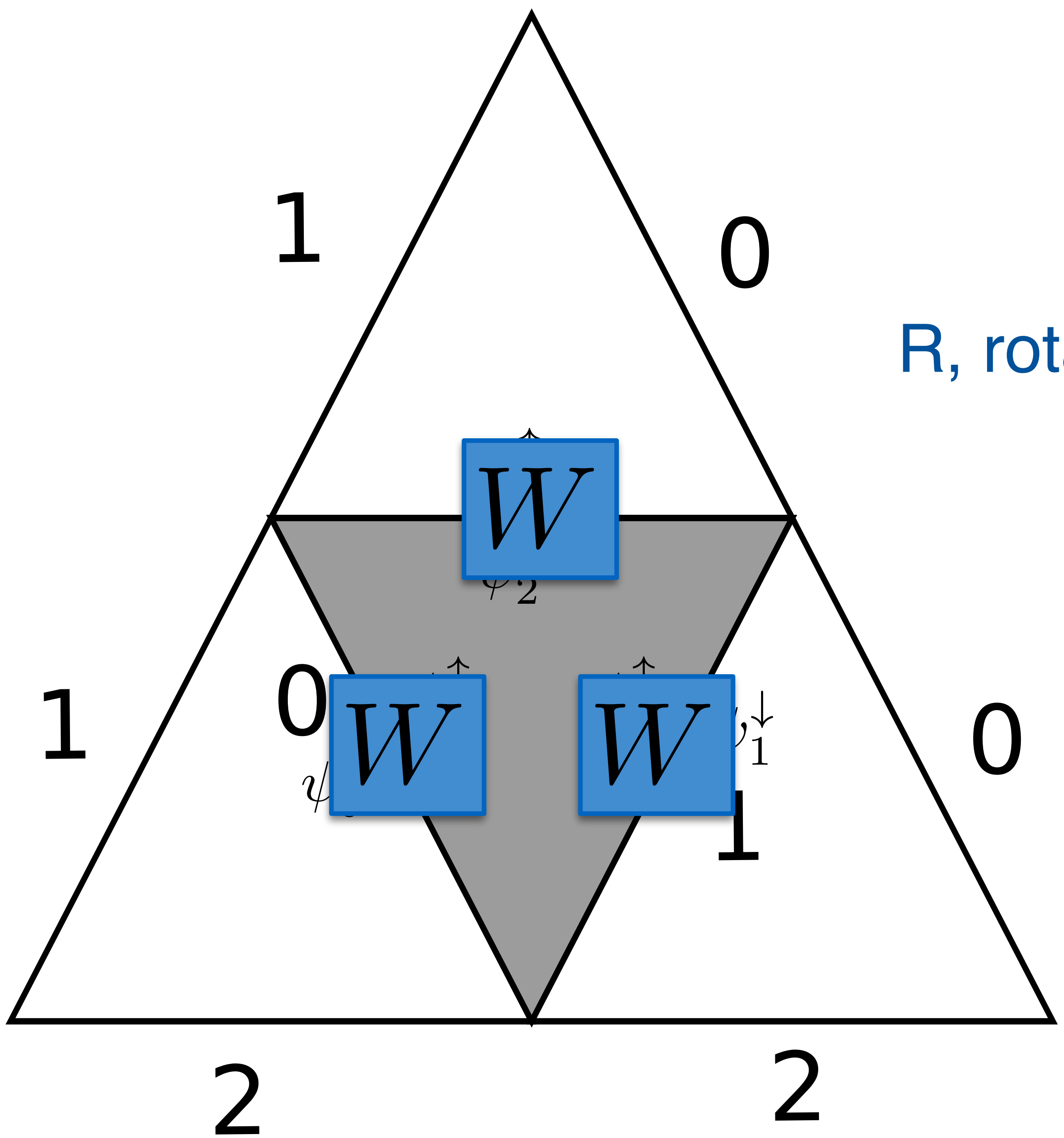
Triangular QW



The first operator, R, rotates every triangle anti-clockwise

III. Hexagonal and triangular QWs

Triangular QW

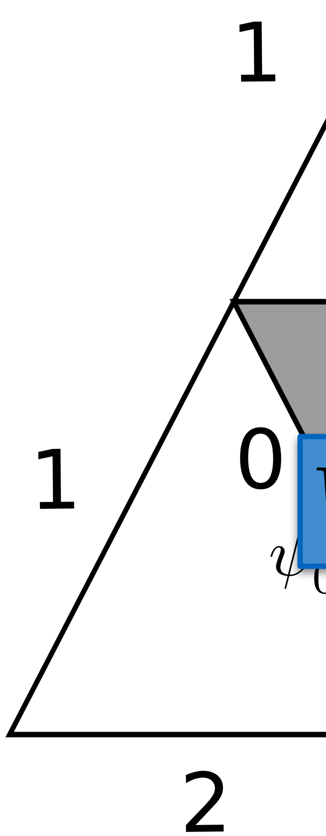


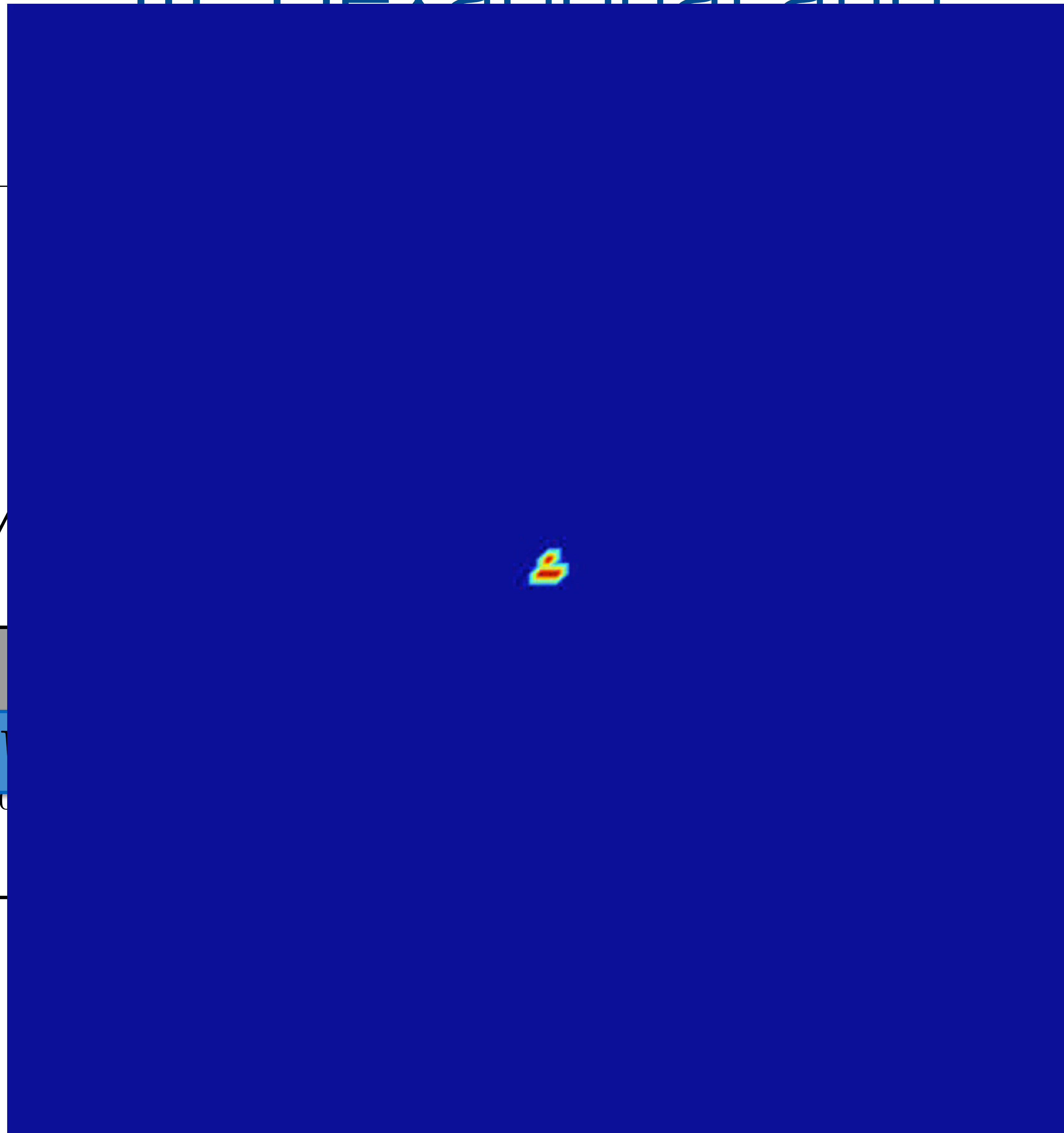
R, rotates every triangle anti-clockwise

W, U(2) matrix

$$W = U_0 S U_0^\dagger M$$

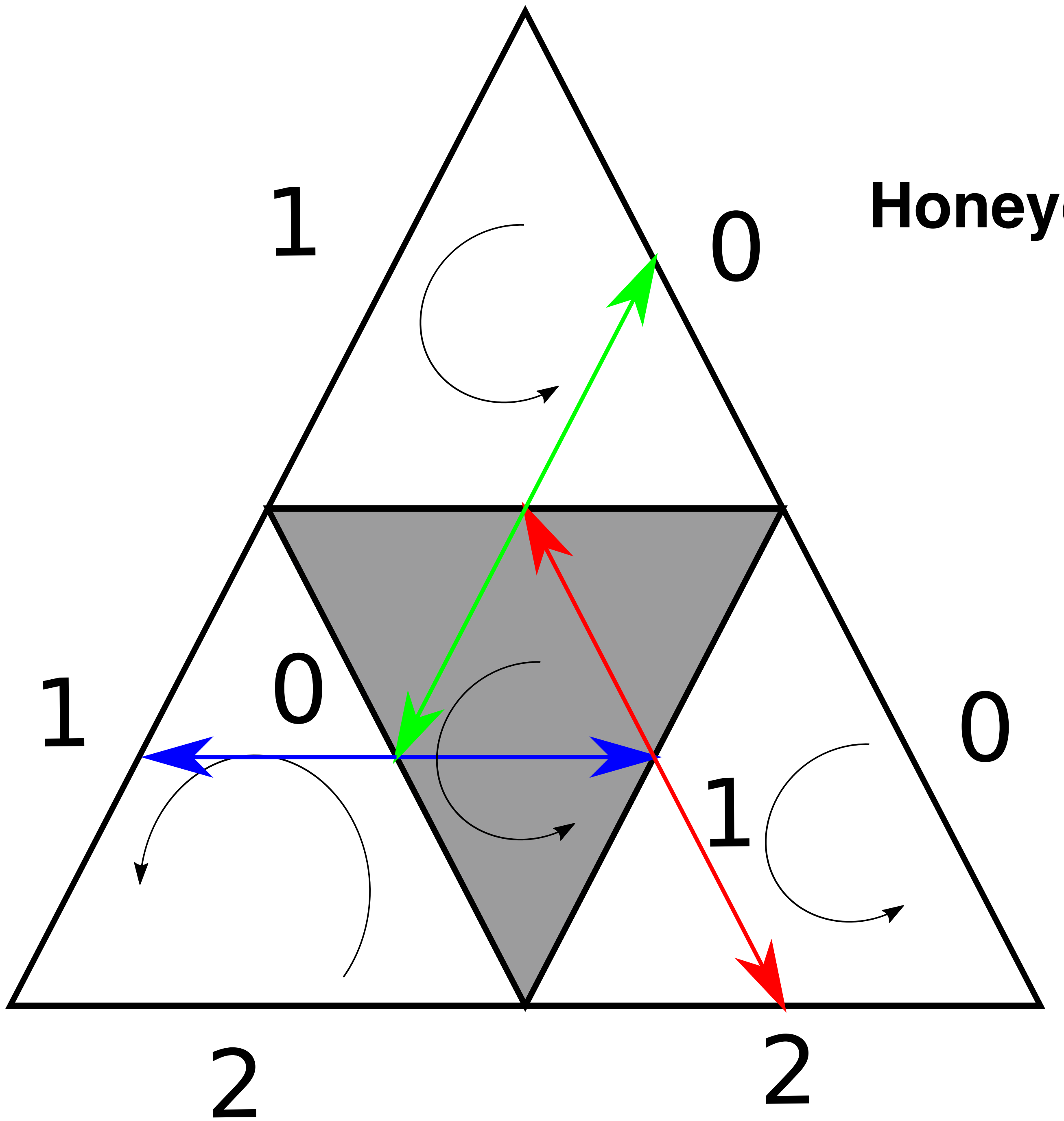
III. Hexagonal and





clockwise

III. Hexagonal and triangular QWs



Honeycomb QW dynamics in a covert way in 3 time-steps

III. Hexagonal and triangular QWs

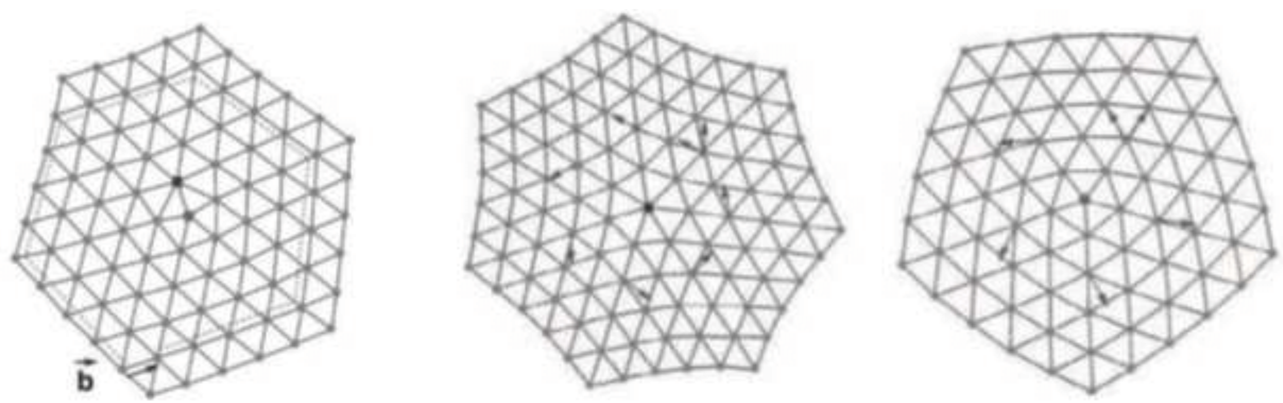
As we expected, we recover again the Dirac eq in the continuous limit

$$\mathrm{i}\partial_t \psi = (p_x \sigma_x + p_y \sigma_y + m \sigma_z)\, \psi$$

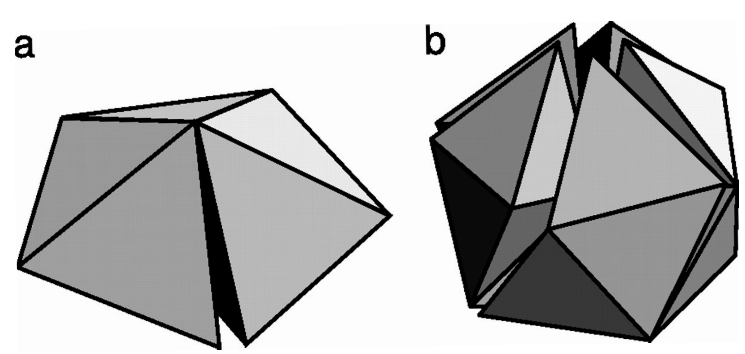
P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. The Dirac equation as a quantum walk over the honeycomb and triangular lattices. Phys. Rev. A 97, 062111

III. Hexagonal and triangular QWs

Defects



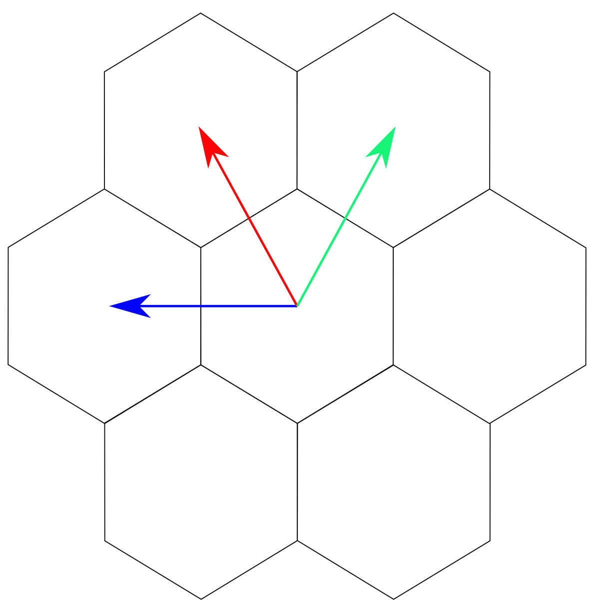
3D



http://www.pnas.org/content/103/28/10612.full

III. Hexagonal and triangular QWs

Understanding propagation in discretized curved spacetime;



in-homogenous transformation

$$\Lambda = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \qquad u_i' = \Lambda u_i$$

$$H_{DC} = \sum_i \pi_i \tau_i'(x, y) + m\sigma_z$$

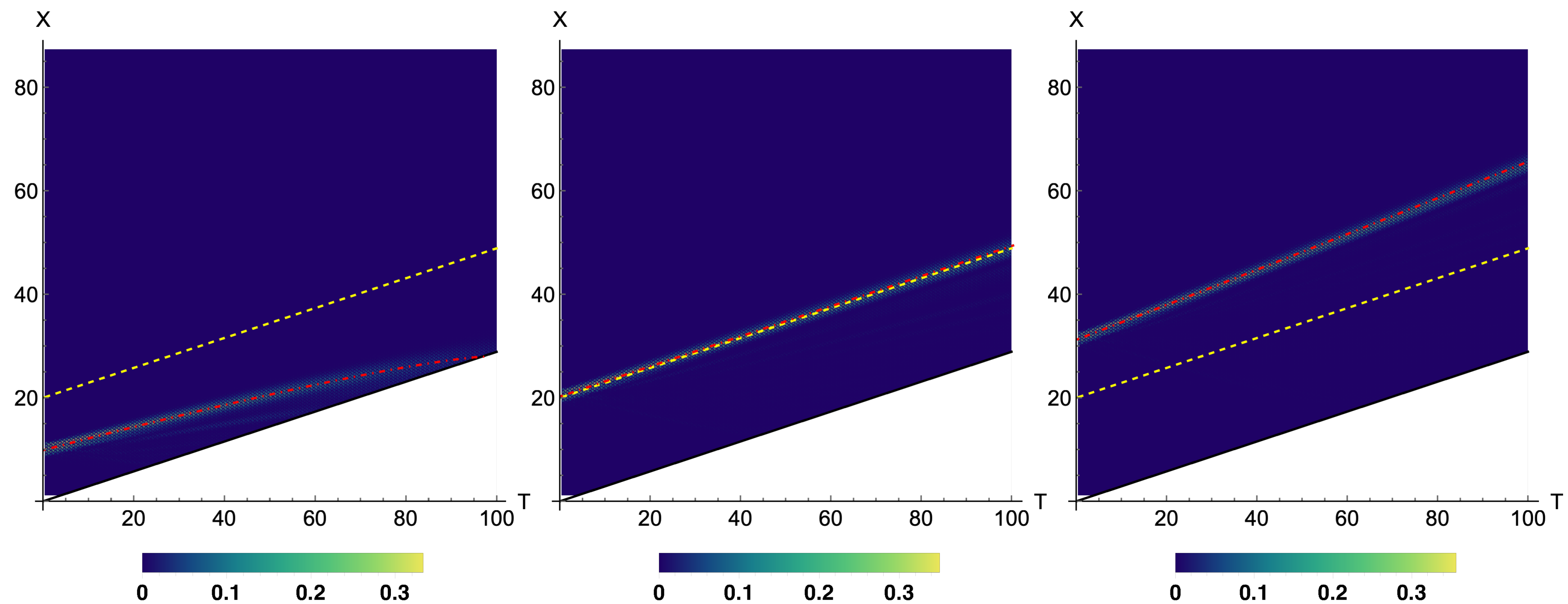
Duality → reabsorbing the deformation in the coin operator

Not possible in the rectangular lattice

III. Hexagonal and triangular QWs

Black hole simulation

$$ds^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \frac{d\rho^2}{1 - \frac{r_s}{r}} - r^2 d\theta^2$$

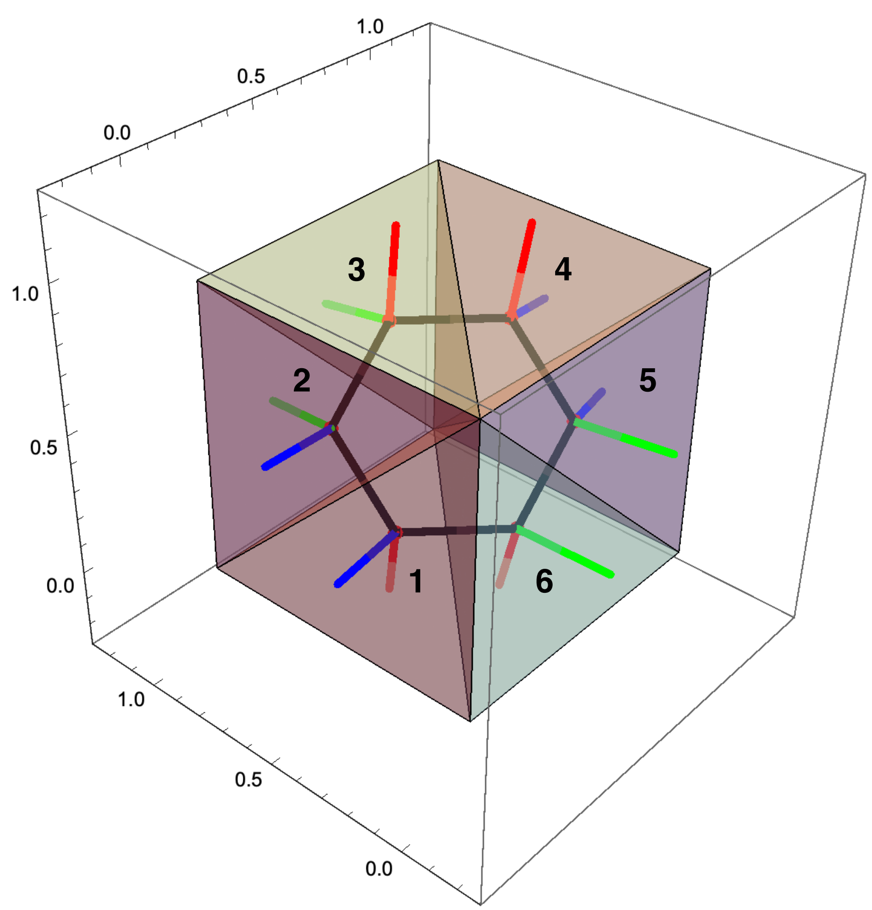


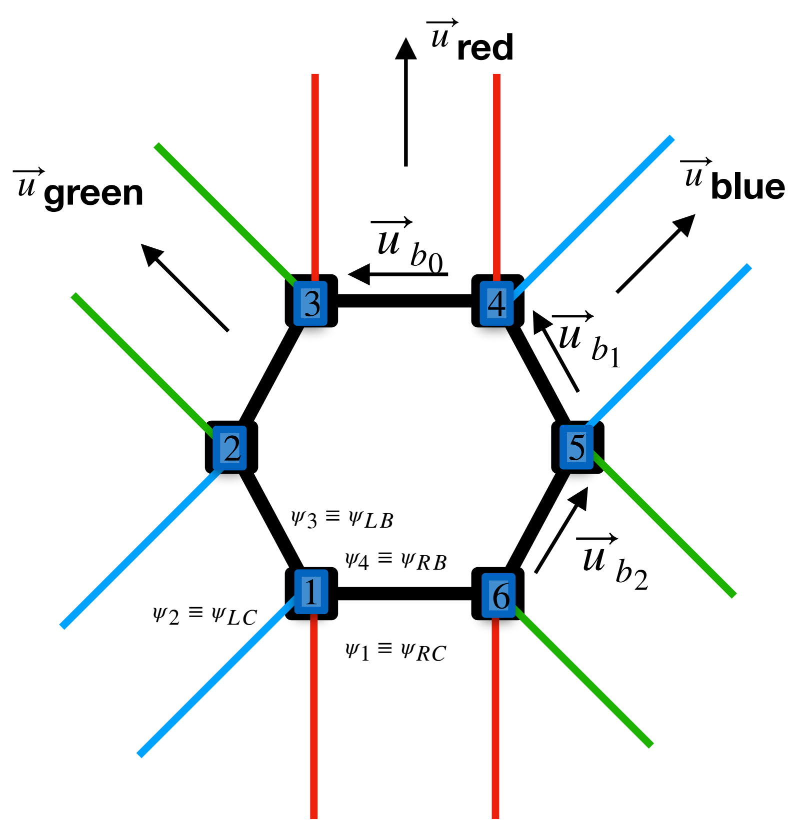
Probability density of a QW in the plane (t,x), compared with the classical geodesic

P. Arrighi, G. D. Molfetta, I. Márquez-Martín, and A. Pérez. From curved spacetime to spacetime-dependent local unitaries over the honeycomb and triangular Quantum Walks. *Scientific Reports* **volume 9**, Article number: 10904 (2019)

III. Hexagonal and triangular QWs

Quantum simulation in 3D QW triangulated?





III. Hexagonal and triangular QWs

Why are we doing that?

Understanding the fermion propagation under discrete space-time

Graphene applications

Antonio Gallerati. "Graphene properties from curved space Dirac equation". In: *European Physical Journal Plus* 134.5 (2019)

Kyriakos Flouris, Sauro Succi, and Hans J. Herrmann. "Quantized Alternate Current on Curved Graphene". In: *Condensed Matter* 4.2 (2019)

Posible applications in quantum algorithms

Thank you!